Exploring Dark Interactions by Destroying Neutron Stars with Dark Black Holes

Joseph Bramante
University of Notre Dame

January 23, Fermilab

1301.0036 JB, Fukushima, Kumar 1310.3509 JB, Fukushima, Kumar, Stopnitzky

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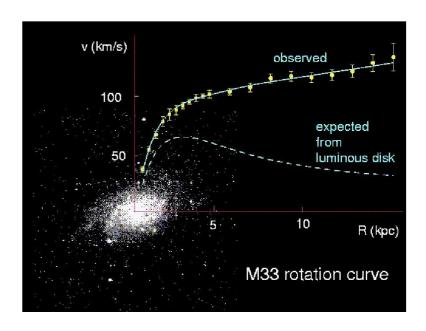
Outline of NS-DM Talk

- Dark matter and dark interactions in the dark dark sector.
- 2. Dynamics of DM collection in neutron stars.
- 3. Neutron star bounds on non-annihilating bosons.
- 4. Neutron star bounds on bosons which self-interact and annihilate. [JB et al. 1301.0036 (PRD)]
- 5. Neutron star bounds on fermions which self-interact and annihilate. [JB et al. 1310.3509 (PRD)]

Rotation curves,

∃ dark massive fields

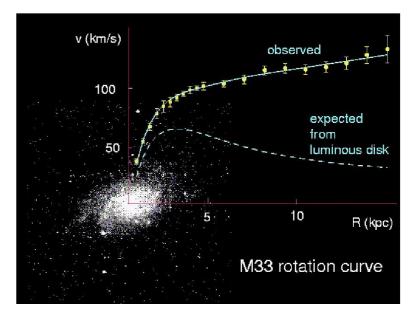
 Rotation curves show galaxies and galactic clusters missing visible mass

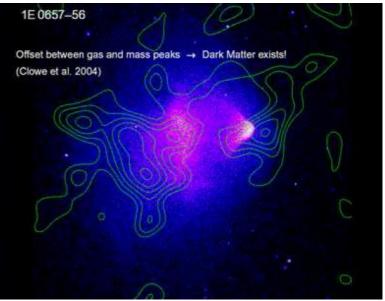


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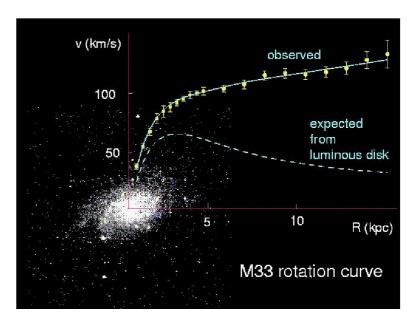


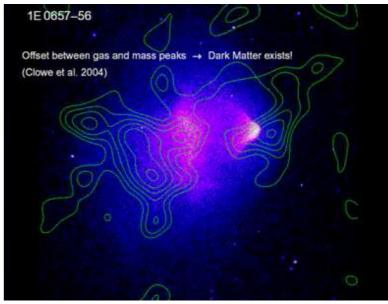


Rotation curves, Bullet clusters, CMB

∃ dark massive fields

- Rotation curves show galaxies and galactic clusters missing visible mass
- Bullet cluster x-ray emitting gas displaced from gravitationally lensed mass distribution
- ΛCDM fits of Planck (WMAP), large scale galaxy distribution, type 1a SN, and BAO data
 - o 20% total energy
 - ~5:1 ratio with VM





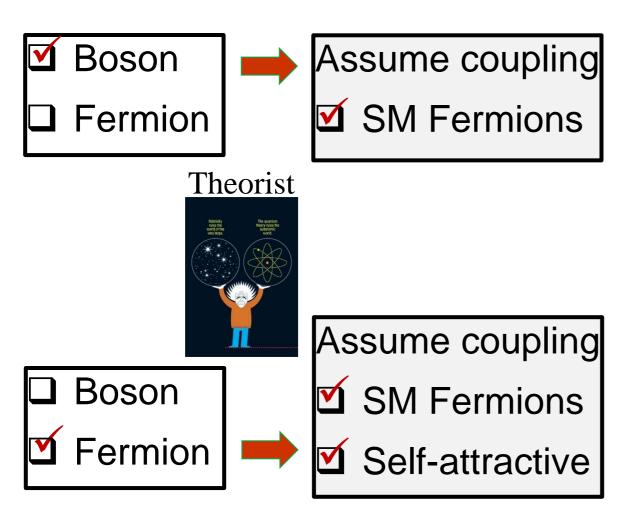
Dark matter has a gravitational interaction, the exciting question is, what other interactions might it have?

Dark Matter Checklist

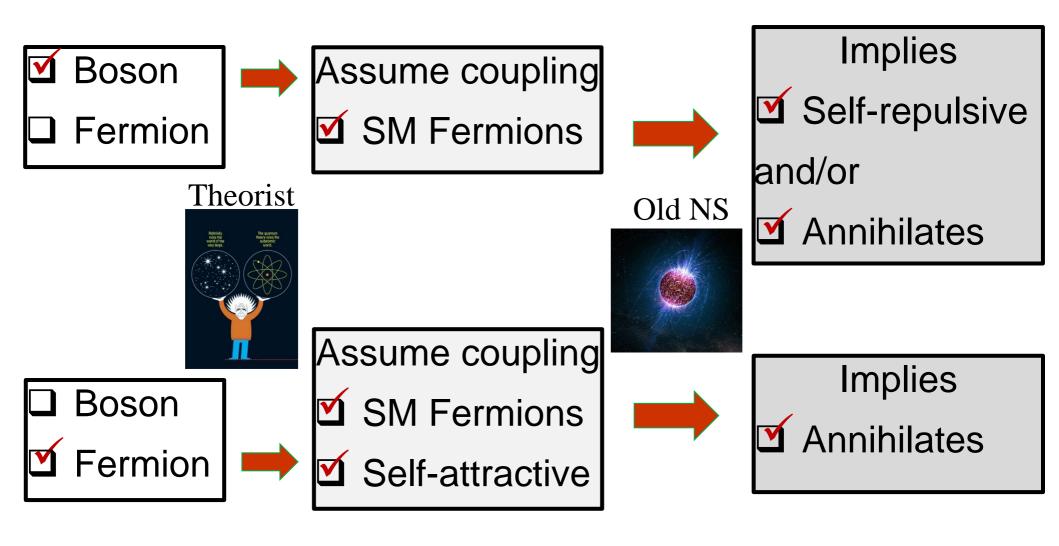
- Mass
- Boson
- □ Fermion
- □ Stability

- Couplings
- ☑ Gravity
- With Standard Model
 - Weak Interactions
 - □ Higgs
 - ☐ Gluon, Photon, W, Z
 - Fermions
- □ Self-coupling
- □ Annihilation
- □ Decay

Old Neutron Stars Imply Relations Among Dark Matter Couplings

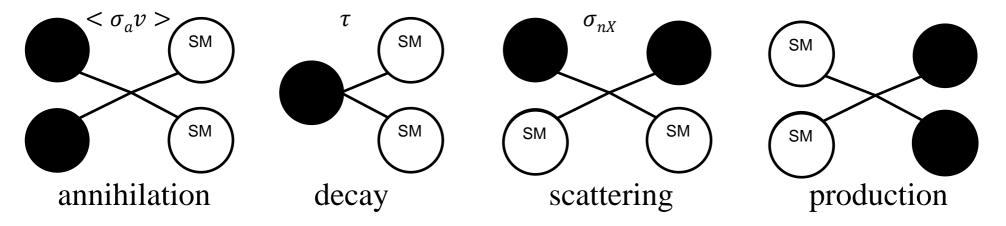


Old Neutron Stars Imply Relations Among Dark Matter Couplings

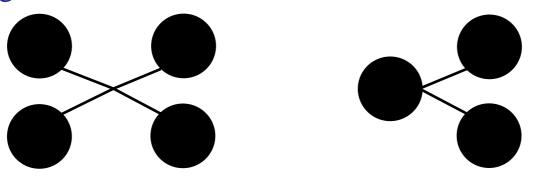


Interactions

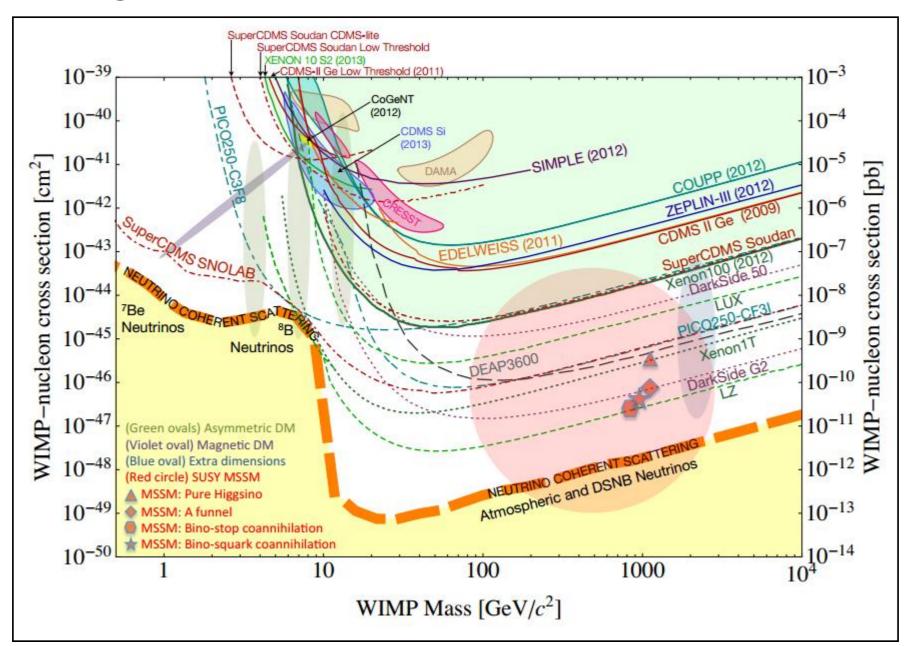
Signals of dark matter at satellites, space stations, colliders, vats of cold inert gas, semiconductors with extremely well understood backgrounds...



Exclusively dark **dark sector** interactions also have phenomenological consequences: halo structure, dwarf galaxy population, relic abundance, separation of gas and mass in bullet clusters...



Signals and Constraints - Direct



Self-interactions, bullet clusters

- Bullet clusters provide an upper bound on dark matter self-interactions.
- Xray-emitting ionized IGM slowed by ram pressure as the subcluster slams through a megacluster.

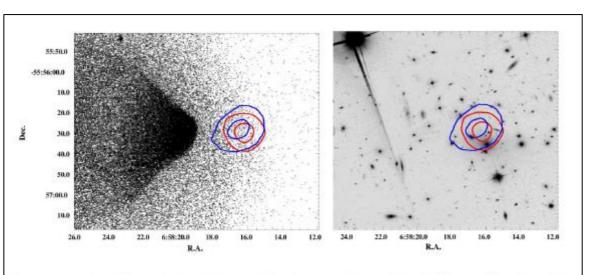


Fig. 2.— Close up of the subcluster bullet region, with the DM (blue) and galaxy (red) centroid error contours overlain. The contours show the 68.3% and 99.7% error regions. The left panel shows the X-ray *Chandra* image, while the right shows the optical *HST* image.

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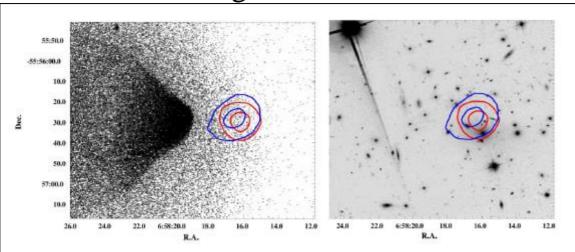


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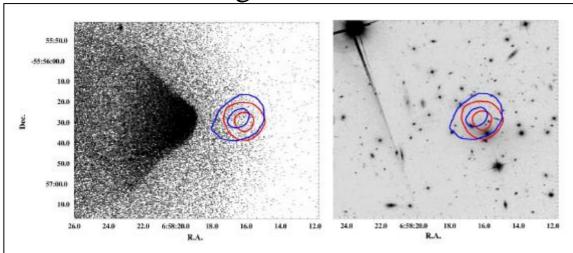


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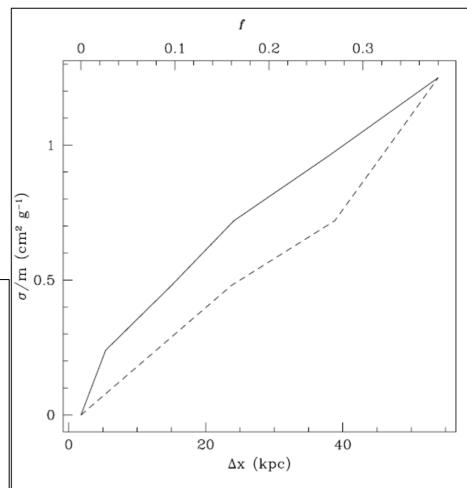


Fig. 5.— The dependence of the subcluster galaxy and total mass centroid offset (Δ x, solid line) and the fractional change in the subcluster M/L ratio (f, dashed line) on σ/m. Based on the values given in Table 2.

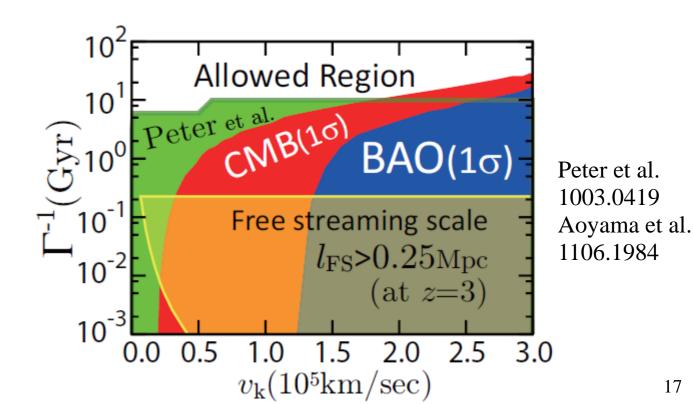
Randall et al. 0704.0261

DM decay – reheating and halo formation

 Constraints on dark matter decay arise from the CMB and the simulation of dark matter galactic halo formation.

DM decay – reheating and halo formation

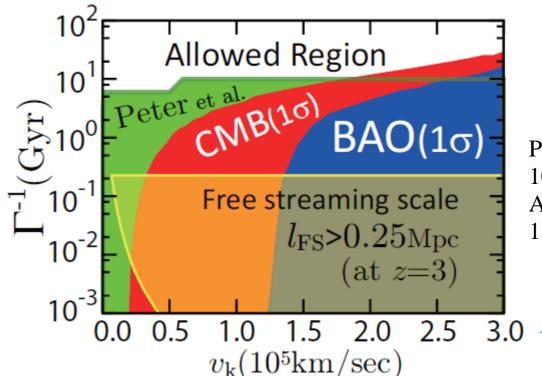
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- Note the green exclusion curve.



DM decay – reheating and halo formation

- Constraints on dark matter decay arise from the CMB and the simulation of dark matter galactic halo formation.
- Note the green exclusion curve.
 - Decaying dark matter imparts a velocity kick (v_k) to decay products. Numerical simulation of the evolution of the mass and density profile of galaxies compared to observed profiles excludes decay rates larger than an inverse gigayear.

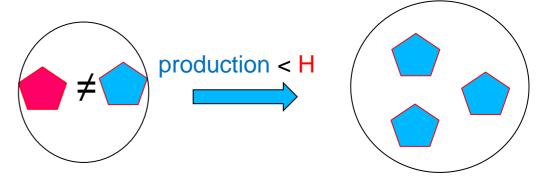




Peter et al. 1003.0419 Aoyama et al. 1106.1984

Relic abundance and Asymmetric DM

- When did dark matter enter the universe?
 - o LCDM fits consistent with **mostly cold, collisionless DM**, freezes out during radiation-dominated expansion of the universe
 - Rate of production of particles less than $H=a'/a \rightarrow leads$ to freeze-out, momentum and number density redshift as the universe expands.

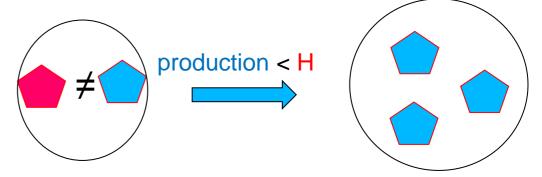


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WIMP miracle

o Particles with weak scale masses (~100 GeV) and weak scale production cross-sections get the right DM abundance

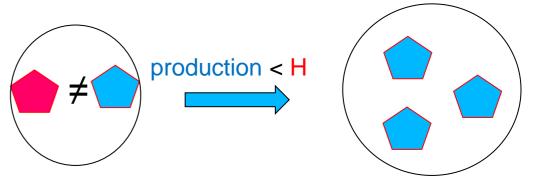


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- WIMP miracle
 - O Particles with weak scale masses (~100 GeV) and weak scale production cross-sections get the right DM abundance
- o However, this is a somewhat arbitrary coincidence.

What about the 5:1 DM:baryon density ratio? Can we tie in the baryon asymmetry?

O Yes! Asymmetric Dark Matter supposes some mechanism produces the baryon asymmetry related to a dark particle-antiparticle asymmetry.



Asymmetric DM can self-annihilate

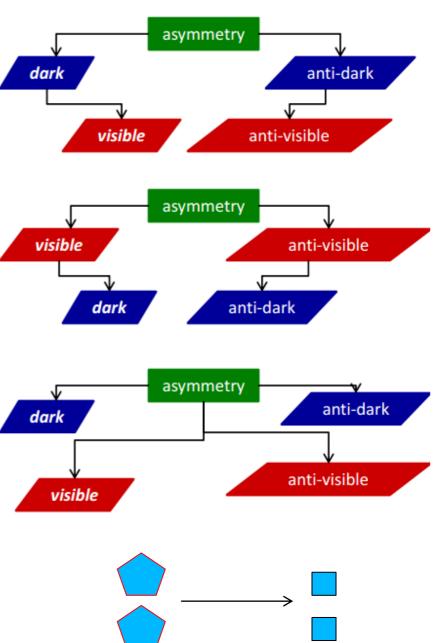
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-ADM freezes out as (anti)particle, doesn't have anything to annihilate with...



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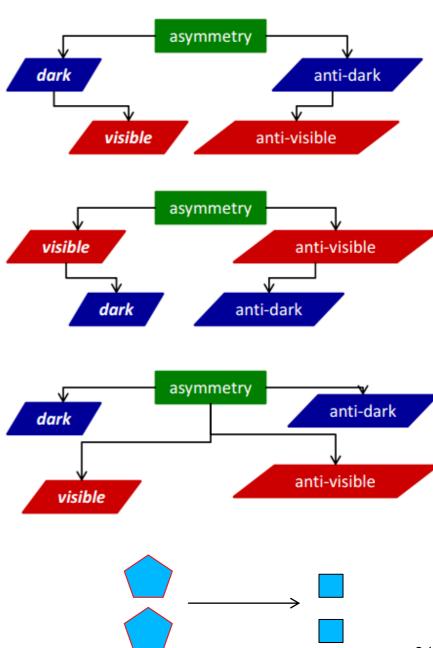
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JB, Kumar, Fukushima 1301.0036 (PRD)

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- -But the **most minimal statement**: asymmetric dark matter freezes out as one part of a complex particle-antiparticle pair this complex, continuous symmetry will remain unbroken under Poincare and charge invariance (and to satisfy Sakharov conditions)
 - -However, the ADM frozen out need not be the lightest particle charged under the complex symmetry...could have an additional \mathbb{Z}_2 symmetry.
 - -Example proton: positive electric charge. If Lepton number and Baryon number were both conserved mod(2), protons could annihilate to e⁺ e⁺.
 - -The key is that this additional annihilation channel must be small enough not to upset freeze-out



JB, Kumar, Fukushima 1301.0036 (PRD)

Nucleon-Scattering Asymmetric DM Must Self-Interact

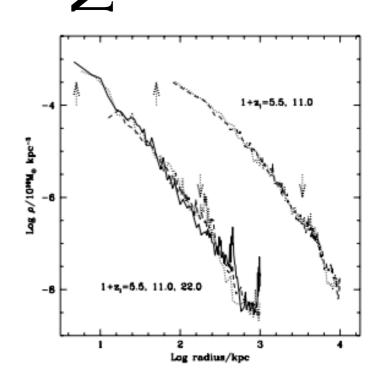
Reasonable UV completions of nucleon contact interactions imply dark matter self-interactions.

DM-nucleon effective operator	$\frac{\alpha_s}{4M_*^2} \chi^\dagger \chi G_{\mu\nu} G^{\mu\nu}$		$rac{m_q}{M_*^2}\chi^\dagger\chiar q q$		$\frac{i}{M_*^2} \chi^{\dagger} \overleftrightarrow{\partial}_{\mu} \chi \bar{q} \gamma^{\mu} q$
DM-nucleon cross-section, $\sigma_{n\chi}$	$3 \cdot 10^{-2} \; \frac{\mu_{n\chi}^2 m_n^2}{m_\chi^2 M_*^4}$		$7 \cdot 10^{-3} \; \frac{\mu_{n\chi}^2 m_n^2}{m_{\chi}^2 M_*^4}$		$\frac{3\mu_{n\chi}^2}{M_*^4}$
possible UV completions	scalar mediation	fermion mediation	scalar mediation	fermion mediation	vector boson mediation
$\chi n \to \chi n$ scattering	x, S , S	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$X \longrightarrow H$ q		γ_D q q
$\begin{array}{c} \chi\chi \to \chi\chi \\ \text{scattering} \end{array}$	x x x x x x x x x x	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	χ χ H χ χ	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	χ χ χ χ χ χ

- Cold collisionless dark matter has been simulated coalescing into DM halos.
- The NFW profile was designed as an analytic formula matched to simulations of cold, collisionless DM forming halos.
- ❖ Note especially that the density of the simulated galaxy halos rises sharply at small radius, (10¹¹ and 10¹⁵ solar mass halos displayed, respectively)

VFW profile

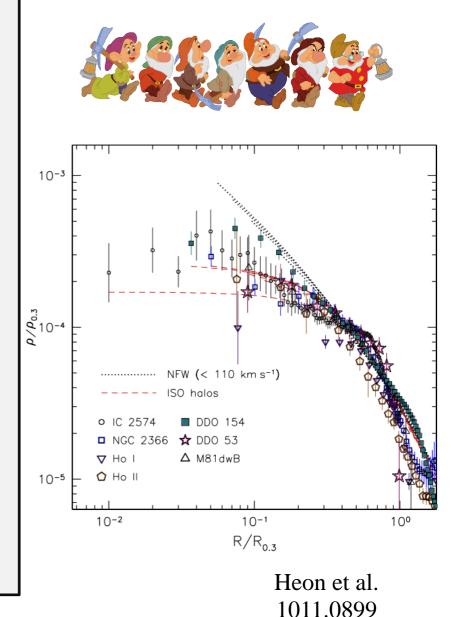




Navarro et al. astro-ph/9508025

Seven dwarves with mined out DM cores

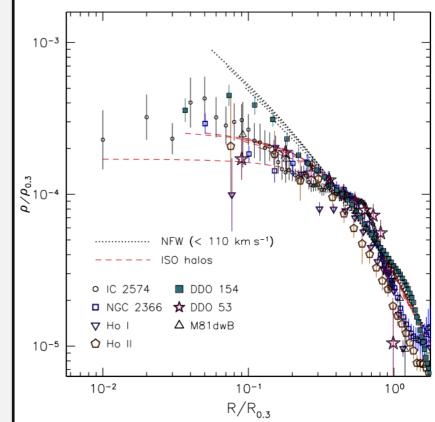
- ➤ 7 dwarf galaxies measured by THINGS do not show a cold, collisionless NFW profile which would cusp in the center (cored shape)
 - Caveat: Baryonic outflow via SN
 - Counter: less luminous galaxies should not experience outflow, but seem to.



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 - Caveat: Baryonic outflow via SN
 - Counter: less luminous galaxies should not experience outflow, but seem to.
- ➤ Also, many simulations suggest that we should have ~50 subhalos in the MW, we see only 12.
 - Caveat: Different models of star formation, subhalos too dim?
 - Counter: "Too big to fail (to form) star subhalos not seen in the Milky Way.





Heon et al. 1011.0899

SIDM cores dwarves, but this is at tension with clusters/spiral galaxies

o Bullet clusters (1000 km/s) and spiral galaxies (200 km/s) constrain the cross-section of dark matter with itself to

$$\sigma/m < 1 \text{ cm}^2/g$$
, $\sigma/m < 1 \text{ cm}^2/g$

But the preferred cross-section to core the dwarf halo (10 km/s) is

$$\sigma/m \sim .1-10 \text{ cm}^2/\text{g}$$

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- This preferred value is very close to the lower bound on DM S-I.
- The scales of observation (dwarf,spiral,cluster) motivate velocitydependent cross-sections -- specific relationships between the mass of the force mediator and dark matter can achieve this.
- Finally, it means that an O(10) more precise measurement of galaxy/mass separation in bullet clusters can test the validity of self-interacting dark matter models.

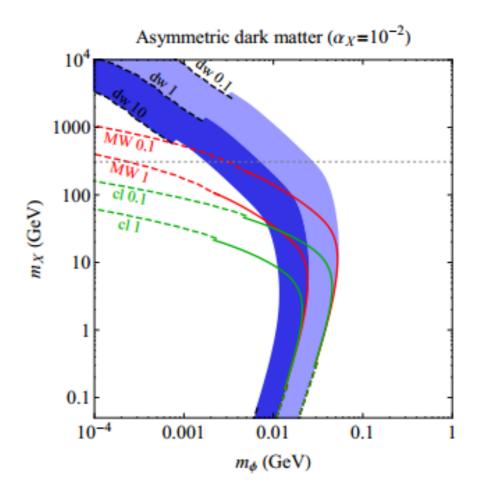
Yukawa SIDM: an expedient solution

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Tulin, Yu, Zurek 1210.0900

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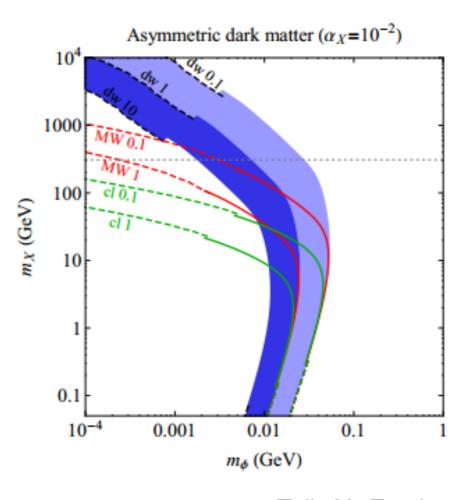
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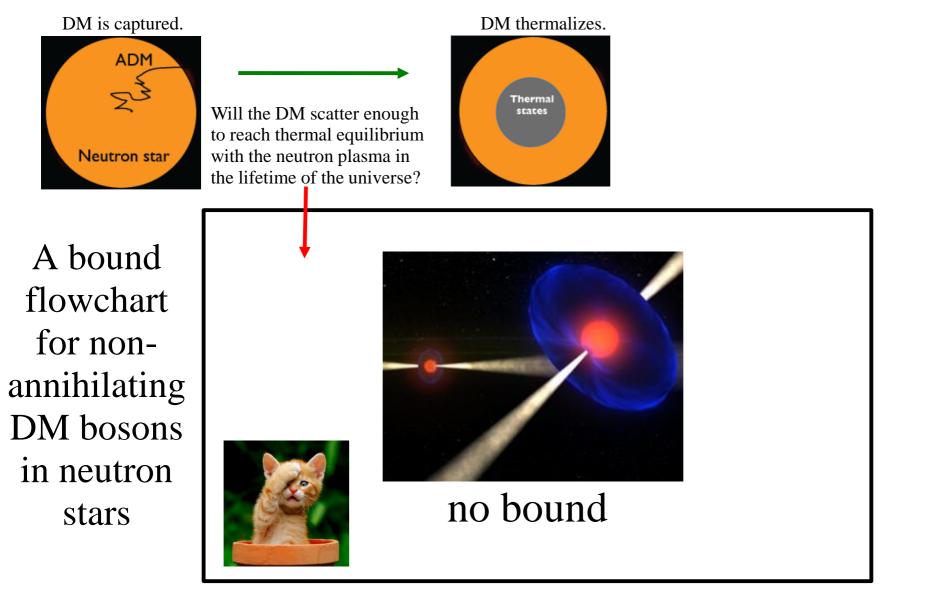
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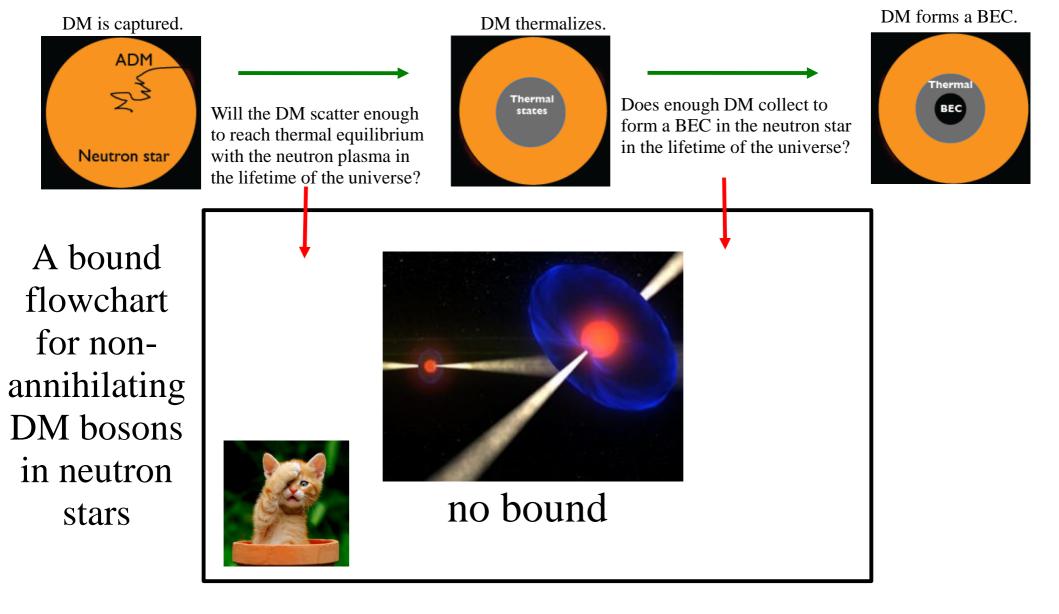
Answer: velocity dependent crosssection provided by light mediator.

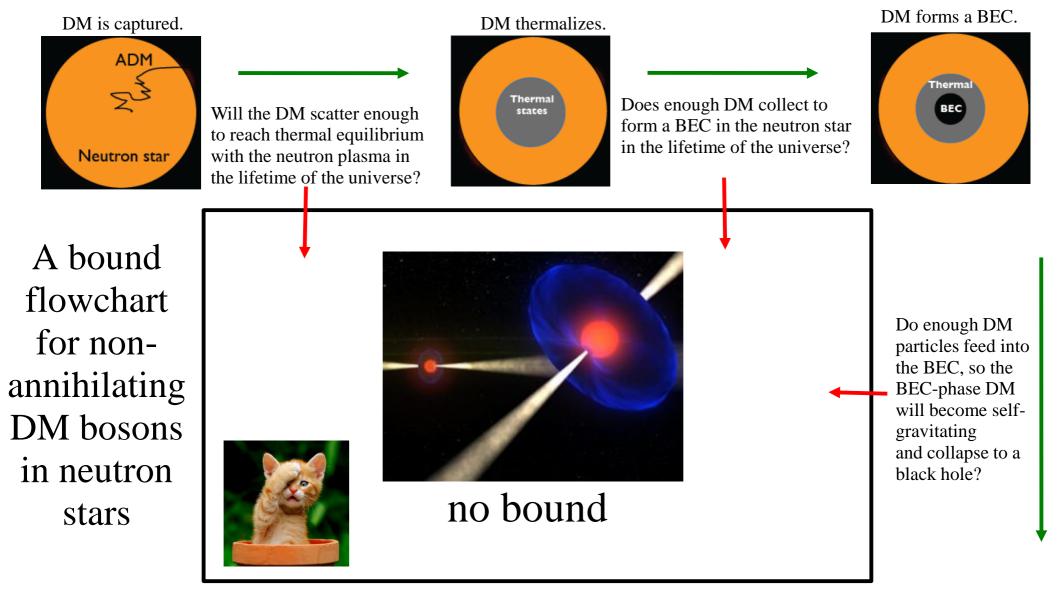
$$\mathscr{L}_{\mathrm{int}} = \left\{ \begin{array}{ll} g_X \bar{X} \gamma^\mu X \phi_\mu & \mathrm{vector\ mediator} \\ g_X \bar{X} X \phi & \mathrm{scalar\ mediator} \end{array} \right.$$



Tulin, Yu, Zurek 1210.0900

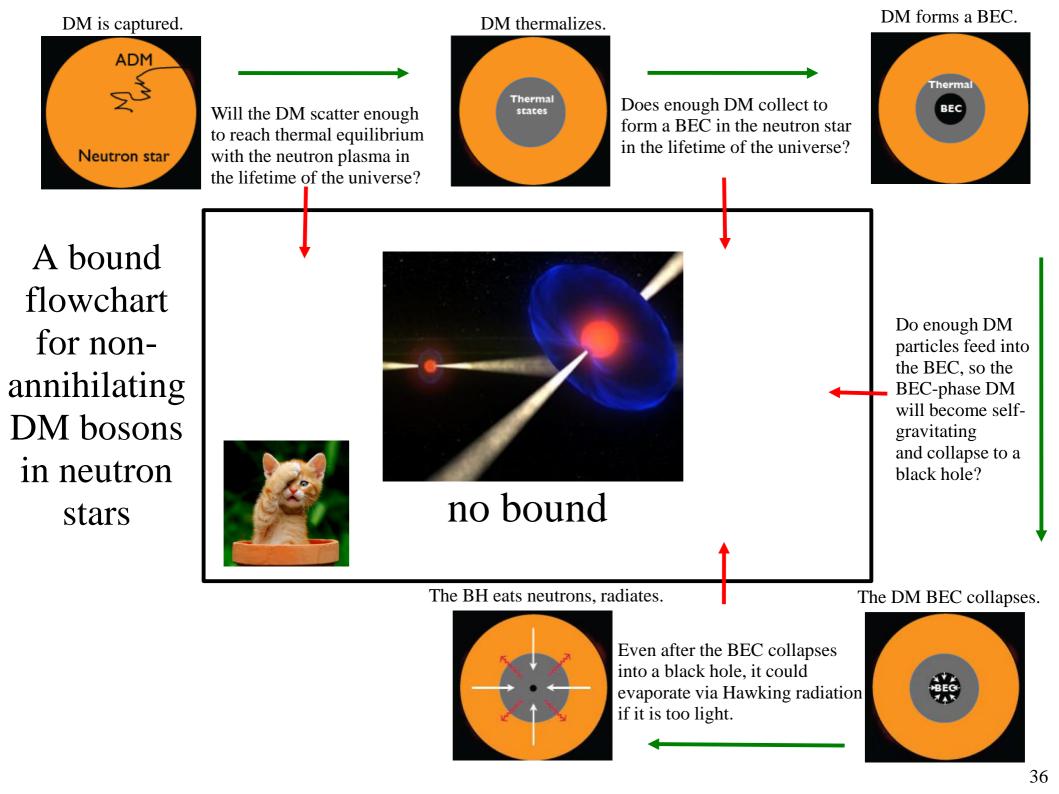


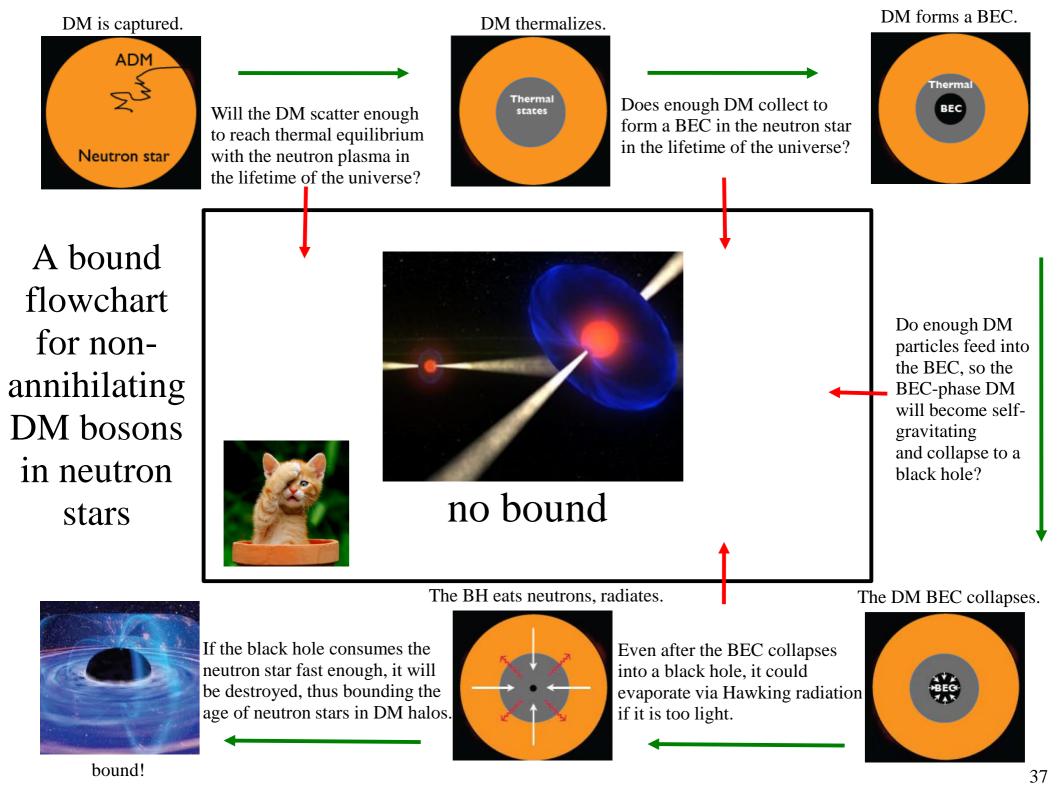




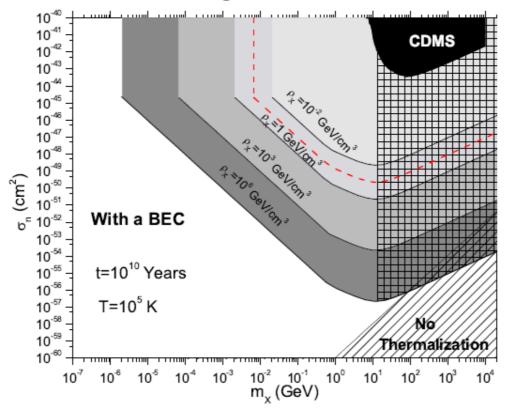
The DM BEC collapses.





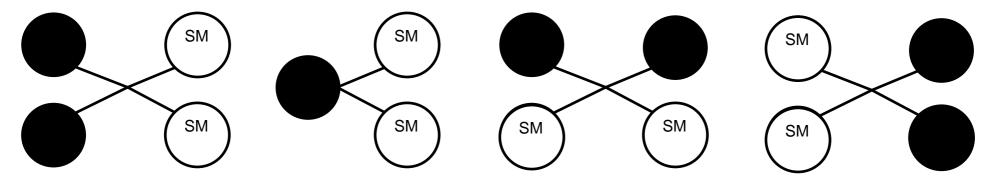


Non-annihilating Bosonic DM bound

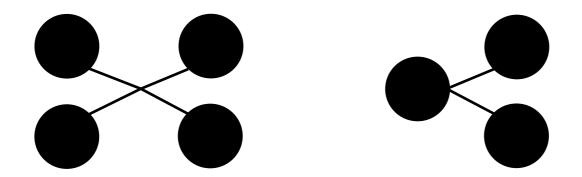


Sam McDermott, Hai-Bo Yu, Kathryn Zurek 1103.5472

- Square hatched = hawking radiation ruins bound
- No thermalization = won't settle to center of NS in 10 Gyr
- Different DM densities → different bounds

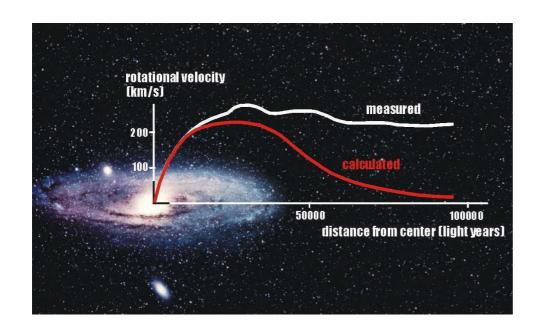


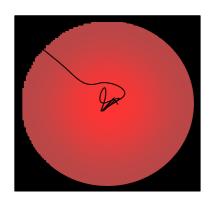
Neutron star bounds on bosonic dark matter that decays, annihilates, and self-interacts

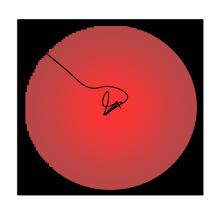


JB, Kumar, Fukushima 1301.0036 (PRD)

- -Neutron stars have been observed surrounded by dark matter particles moving at ~220 km/s.
- -If these dark matter particles scatter off neutrons, dark matter will collect in the neutron star.

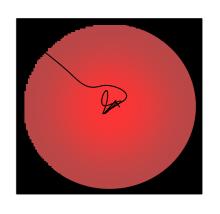






- There is a saturation point for the DM-(neutron star) cross-section.
- Above a certain DM-nucleon cross-section, the probability for a DM particle to scatter if it falls into the neutron star's gravitational well approaches unity.

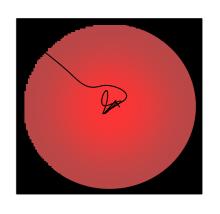
$$P = 1 - \exp\left[-\int \eta_n \sigma_{nX} dl\right]$$



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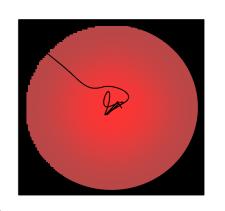
• So for capture, implied cross-section will be the maximum of σ_{nX} and σ_{sat} .



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- So for capture, implied cross-section will be the maximum of σ_{nX} and σ_{sat} .
- A number of additional factors apply:
- Full geodesic path of the DM particle in a full GR treatment.
- Pauli blocking of degenerate neutrons for $m_x < m_n$.
- Pauli blocking in turn alters the DM-NS scattering saturation.

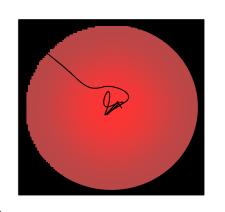


The capture rate is approximately (for saturated cross-section)

$$C_X \sim 2.3 \times 10^{45} \; {\rm Gyr^{-1}} \; \left(\frac{{\rm GeV}}{m_X} \right) \left(\frac{\rho_X}{10^3 \; {\rm GeV/cm^3}} \right)$$

and for
$$m_x < 1 \text{ GeV}$$
, $C_X \sim 3.4 \times 10^{45} \text{ Gyr}^{-1} \left(\frac{\rho_X}{10^3 \text{ GeV/cm}^3} \right)$

Ignore the β and f terms. They are kinematic and saturation terms often equal to unity. (Ok, I removed the β and f terms to help you ignore them, but keep in mind these are there and must be accounted for...)



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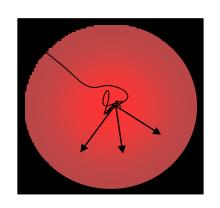
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Compare capture rate to the Chandresekhar black hole collapse number for fermions and bosons

Fermions:
$$N_{chand} = 10^{57} (GeV/m_x)^3$$
 —no bound

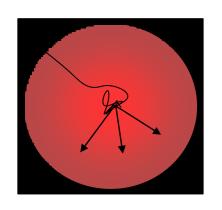
Bosons:
$$N_{chand} = 10^{38} (GeV/m_x)^2$$
 -stringent bound



Decaying dark matter does not significantly alter capture

The number of collected dark matter particles as a function of the age of the neutron star t_{ns} and decay time τ is given by

$$N_{acc}^{(decay)} = C_X \tau \left(1 - e^{-t_{ns}/\tau}\right)$$

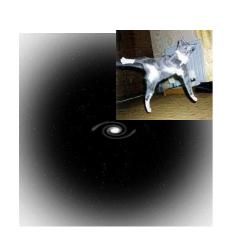


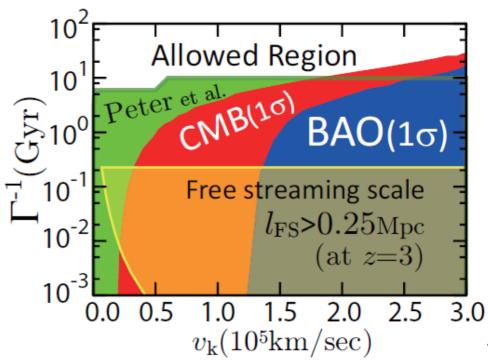
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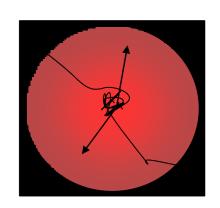
$$N_{acc}^{(decay)} = C_X \tau \left(1 - e^{-t_{ns}/\tau}\right)$$

Note that the green bound stipulates τ is greater than 10 Gyr – so dark matter collected by a 10 Gyr old neutron star will at best be altered by an O(1) factor.





Annihilating dark matter significantly alters DM capture

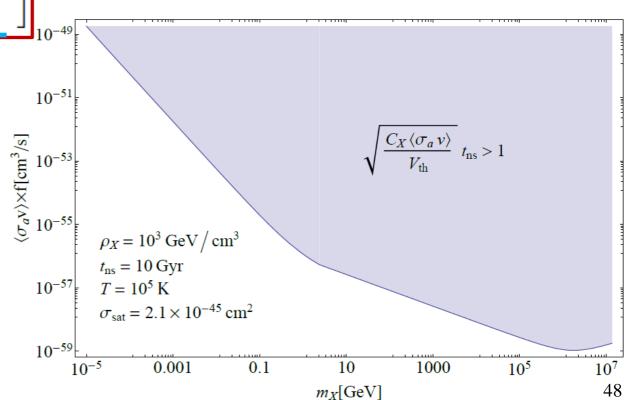


The collection rate of annihilating dark matter particles is

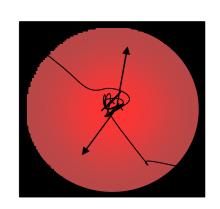
$$\frac{dN_{acc}}{dt} \approx C_X - \frac{\langle \sigma_a v \rangle N_{acc}^2}{V_{th}}$$

$$\rightarrow N_{acc} \approx \sqrt{\frac{C_X V_{th}}{\langle \sigma_a v \rangle}} \operatorname{Tanh} \left[\sqrt{\frac{C_X \langle \sigma_a v \rangle}{V_{th}}} t_{ns} \right]$$

In the limit that dark matter annihilation and capture reach equilibrium, the Tanh term approaches 1.



Annihilating dark matter significantly alters DM capture

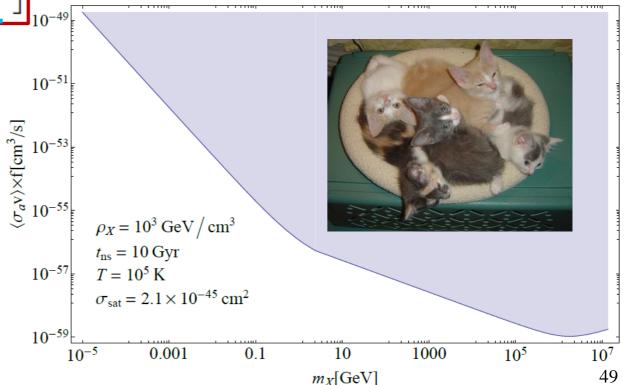


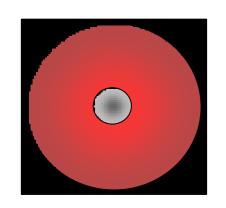
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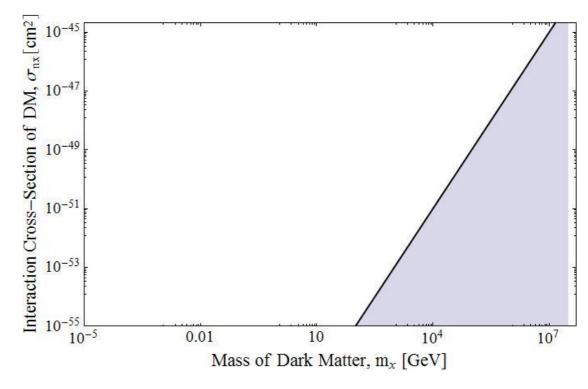


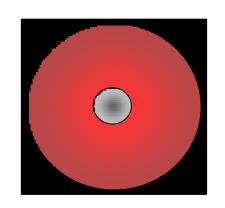
Thermalization

In order for the dark matter to settle into a thermalized core at the center of the neutron star, it must scatter enough to reach thermal equilibrium with the neutrons in the star. For higher masses and lower cross-sections with neutrons, the time required for this can be longer than the age of the universe.

$$r_{th} = 240 \text{ cm} \left(\frac{T}{10^5 \text{ K}} \cdot \frac{\text{GeV}}{m_X} \right)^{1/2}$$

$$t_{th} = 5.4 \times 10^{-6} \text{years} \left(\frac{m_X}{\text{GeV}}\right)^2 \left(\frac{10^5 \text{ K}}{T}\right) f^{-1}$$



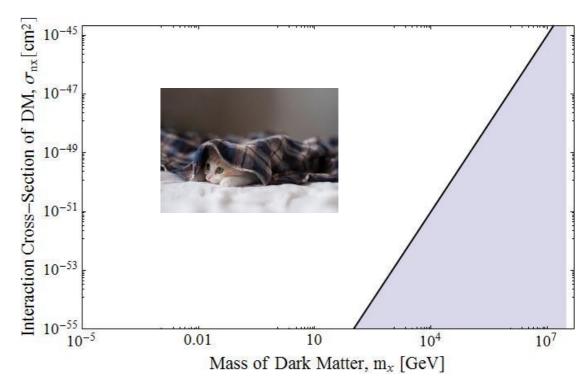


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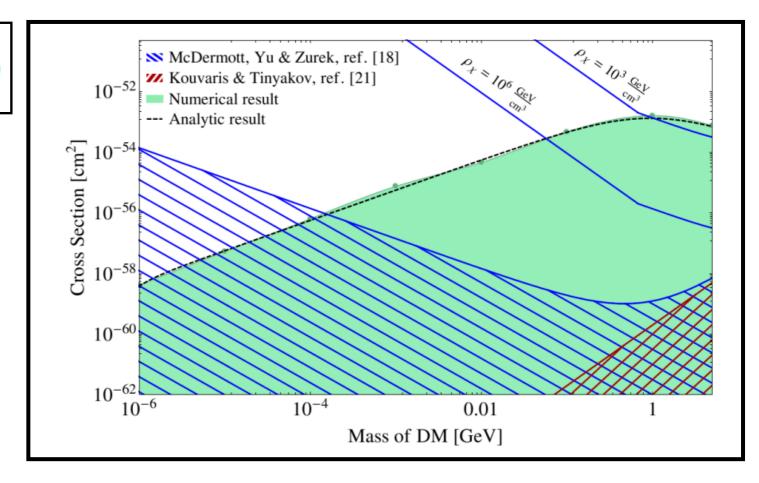


Recent Steps Towards Precision DM-Neutron Star Thermalization

$$\mathcal{L}_{int} = \tilde{G}\ell_{\mu} \left(j_{V}^{\mu} + \alpha j_{A}^{\mu} \right)$$

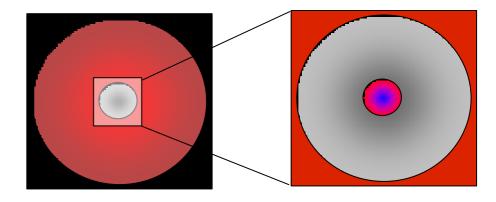
$$\ell_{\mu} = \partial_{\mu} \chi^{\dagger} \chi - \chi^{\dagger} \partial_{\mu} \chi$$

- This treatment assumed a 100 GeV heavy mediator and axial/vector SM coupling to dark matter.
- In a full treatment, thermalization time may be shorter for heavier DM.



Bridget Bertoni, Ann E. Nelson, Sanjay Reddy 1309.1721 (PRD)

BEC formation

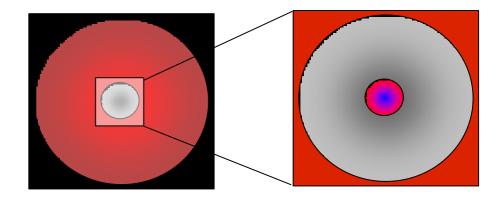


The density at the center of the neutron star is great enough for bosonic dark matter to begin forming a BEC when it reaches a critical number density. The temperature for BEC formation yields a number beyond which all collected DM will condense at neutron star temperature,

$$N_{BEC} = \zeta \left(\frac{3}{2}\right) \left(\frac{m_X T}{2\pi}\right)^{3/2} \left(\frac{4\pi r_{th}^3}{3}\right) \approx 10^{36} \left(\frac{T}{10^5 \text{ K}}\right)^3$$

JB, Kumar, Fukushima 1301.0036 (PRD)

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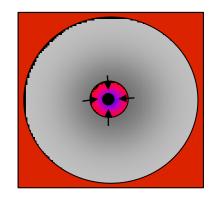
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and the radius of this BEC will be (equating the kinetic energy of ground-state bosons with the gravitational potential energy).

$$r_c = \left(\frac{3}{8\pi G m_X^2 \rho_b}\right)^{1/4} = 1.5 \times 10^{-4} \text{ cm} \left(\frac{\text{GeV}}{m_X}\right)^{1/2}$$

JB, Kumar, Fukushima 1301.0036 (PRD)

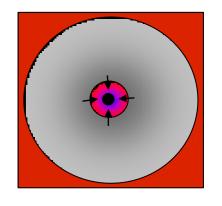
Black hole formation



If the energy of the bosonic dark matter is minimized for an arbitrarily small radius, it will collapse into a black hole. The gravitational potential in the neutron star is

$$E \sim \frac{1}{r} - \frac{Gm_X^2 N_{DM}}{r} + \frac{2\pi G\rho_b m_X r^2}{3} \longrightarrow N_{chand} \sim m_{\rm Pl}^2 / m_X^2$$

Black hole formation



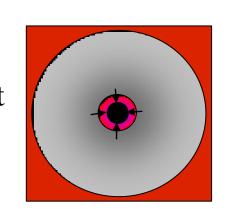
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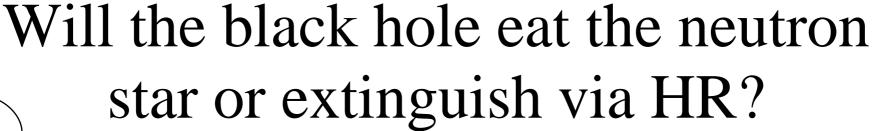
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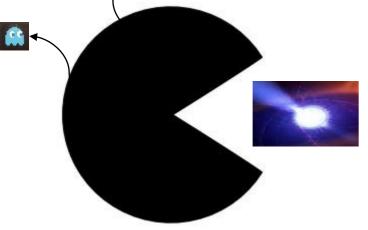
...with self-interactions

Repulsive self-interactions via a $\lambda |\phi|^4$ coupling yield a different limit for collapse:

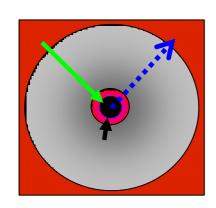
$$N_{chand} = \frac{2m_{\rm Pl}^2}{\pi m_X^2} \left(1 + \frac{\lambda}{32\pi} \frac{m_{\rm Pl}^2}{m_X^2} \right)^{1/2}$$







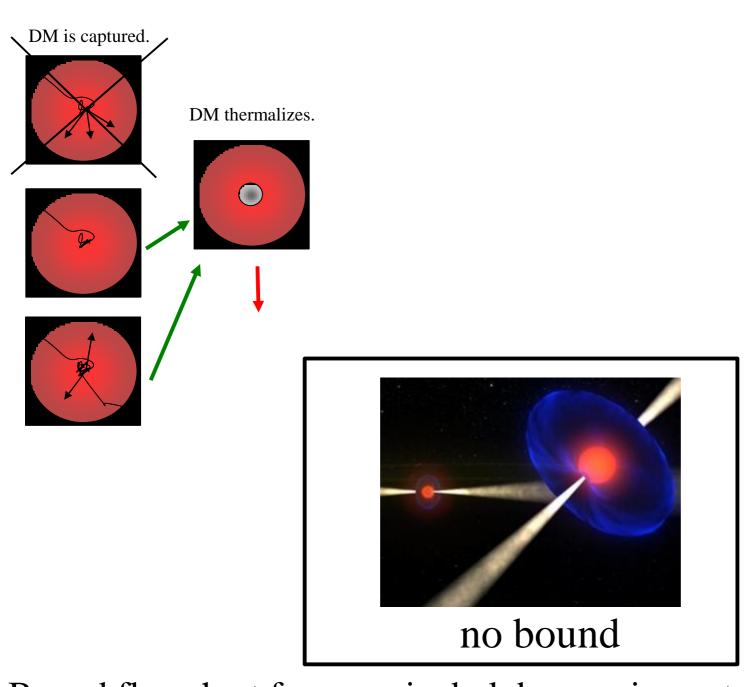
The total mass evolution of the black hole can be approximated with Bondi accretion, Hawking radiation, and infall of dark matter from the dark matter particles that will reform a BEC as more dark matter enters the neutron star.



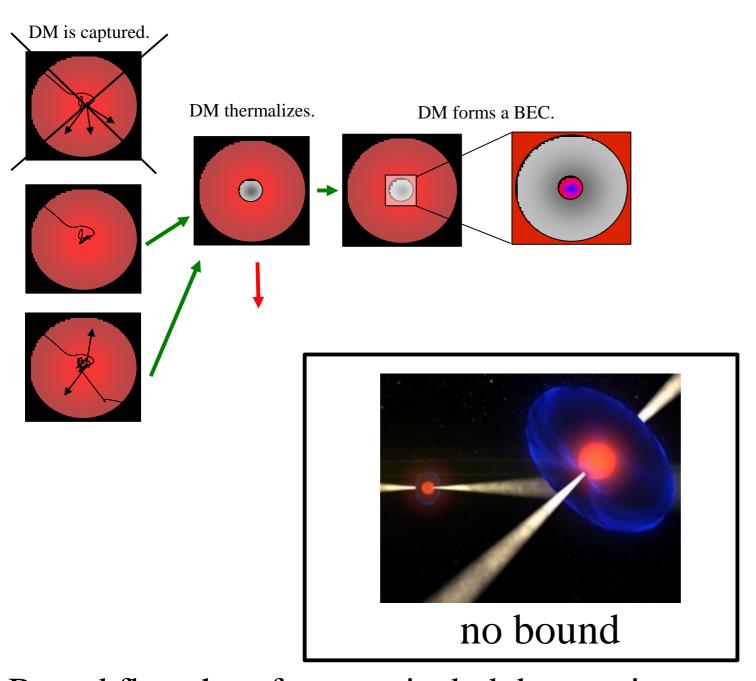
$$\frac{dM_{bh}}{dt} = \frac{4\pi\rho_b(GM_{bh})^2}{v_s^3} + \left(\frac{dM_{bh}}{dt}\right)_{DM} - \frac{1}{15360\pi(GM_{bh})^2}$$

The scattering impact parameter is small compared with the BEC radius → dark matter falls into the star at about the rate it is collected.

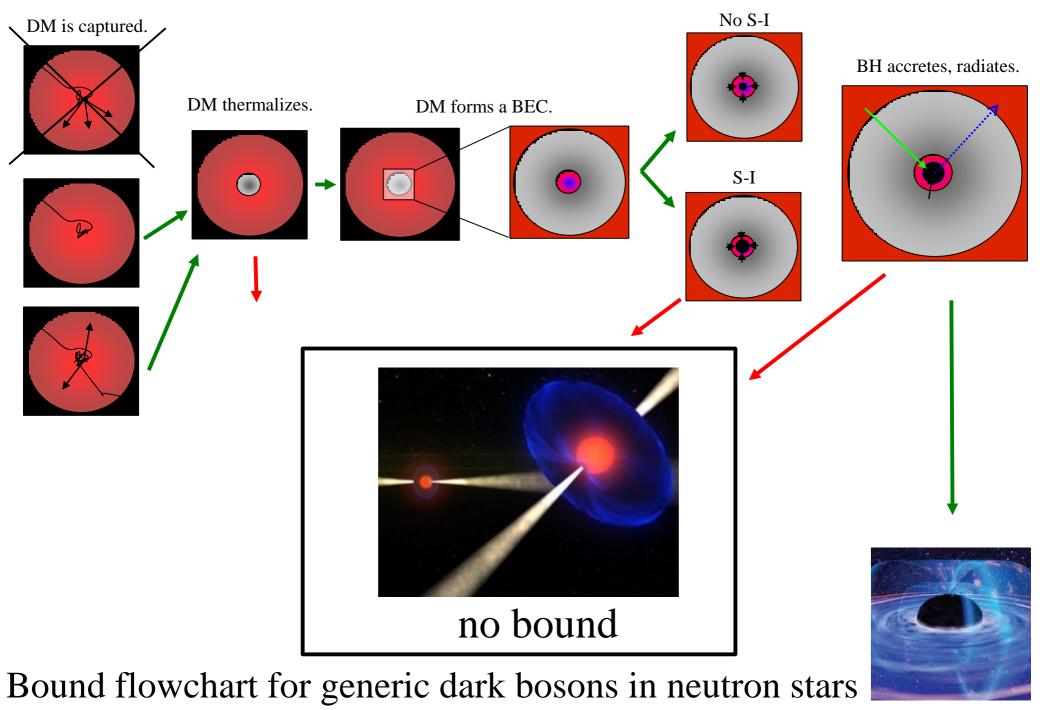
 $b_{infall} \sim 4r_c$



Bound flowchart for generic dark bosons in neutron stars JB, Kumar, Fukushima 1301.0036 (PRD)

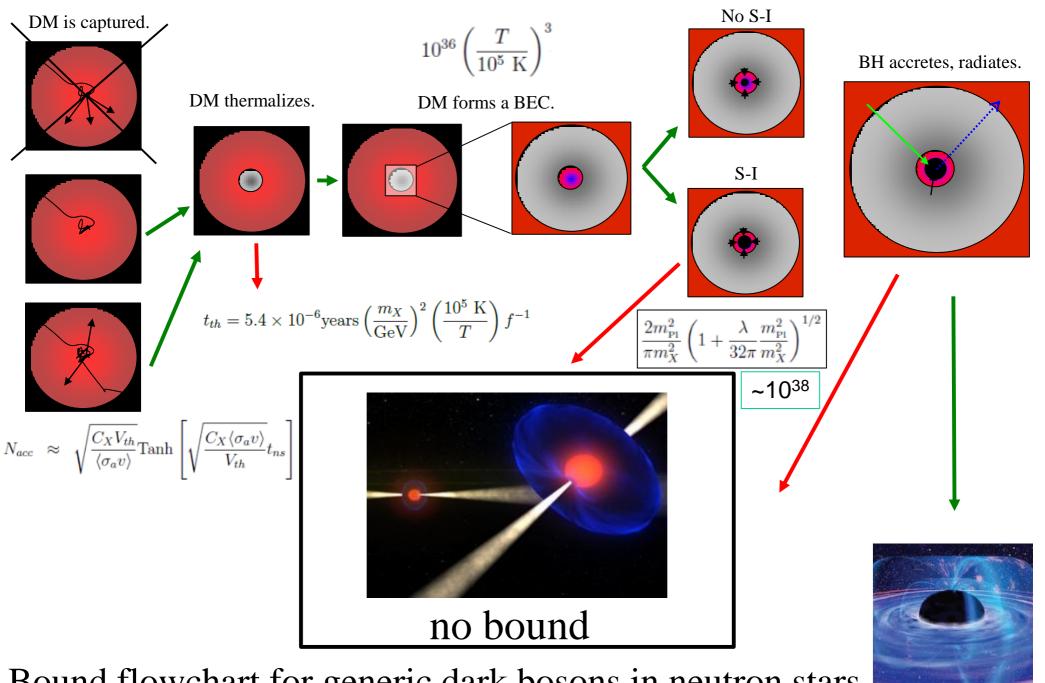


Bound flowchart for generic dark bosons in neutron stars JB, Kumar, Fukushima 1301.0036 (PRD)



JB, Kumar, Fukushima 1301.0036 (PRD)

bound!



Bound flowchart for generic dark bosons in neutron stars JB, Kumar, Fukushima 1301.0036 (PRD)

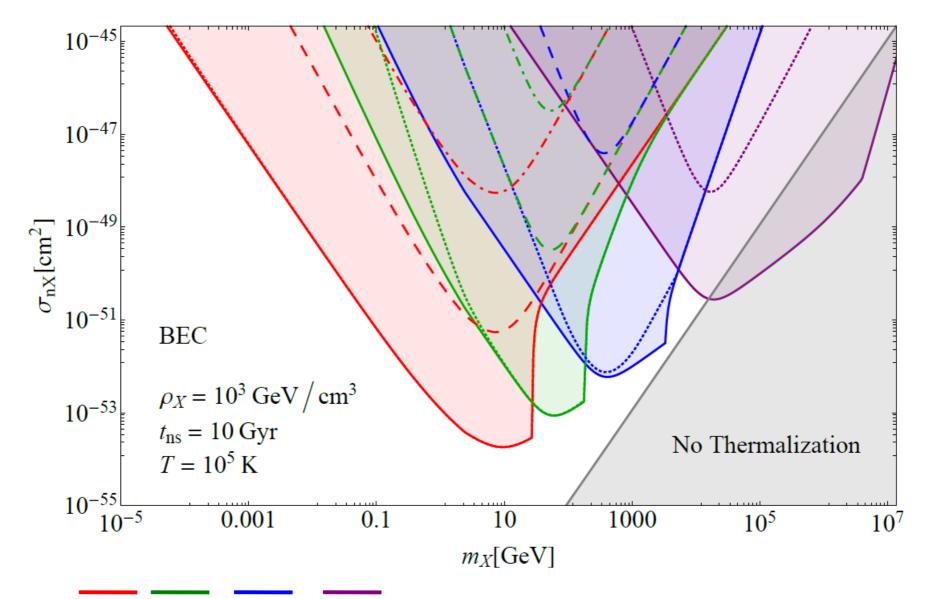
bound!

This all boils down to

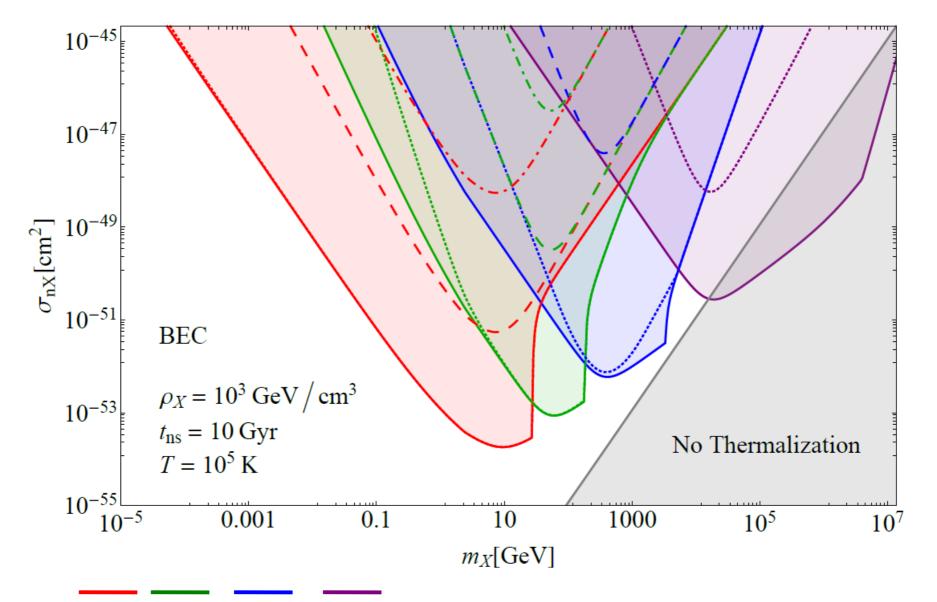
$$N_{acc}(\sigma_{nX}, m_X, \langle \sigma_a v \rangle, \rho_X, t_{ns}, T) > N_{BHforms}(m_X, \lambda, T),$$

$$\frac{dM_{BH}}{dt} \Big|_{M_{BH}=M_{BHi}} > 0.$$

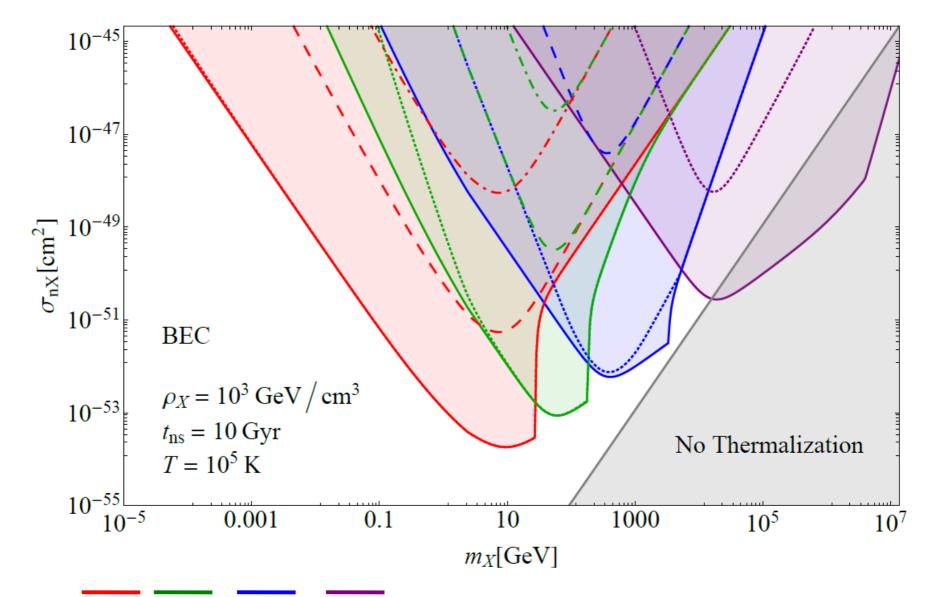
If both statements are true, there is a bound on dark matter.



•
$$\lambda = 0 \ 10^{-30} \ 10^{-25} \ 10^{-15}$$



• $\lambda = 0 \ 10^{-30} \ 10^{-25} \ 10^{-15} \ [\sigma_{xx} \sim 0, \ 10^{-118}, 10^{-98}, 10^{-58} \ cm^2]$

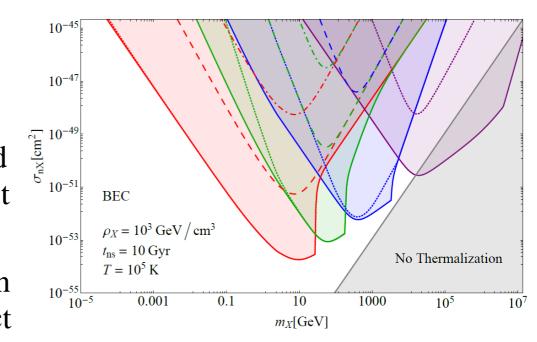


•
$$\lambda = 0 \ 10^{-30} \ 10^{-25} \ 10^{-15} \ [\sigma_{xx} \sim 0, \ 10^{-118}, 10^{-98}, 10^{-58} \ cm^2]$$

•
$$\langle \sigma_a v \rangle = 0 | 10^{-50} | 10^{-45} | 10^{-42} | cm^3/s$$

Boson Bounds

- Bosonic dark matter with small repulsive self-interaction is bounded at higher masses.
- Any dark matter bosons discovered at detectors in the next decade must self-interact or annihilate.
- ...however the required annihilation cross-section is smaller than indirect searches can currently probe (1/100 of a picobarn).
- The required small annihilation cross-section has consequences for freeze-out dynamics and symmetries in model building.



•
$$\lambda = 0 \ 10^{-30} \ 10^{-25} \ 10^{-15}$$

• $\langle \sigma_a v \rangle = 0 \ 10^{-50} \ 10^{-45} \ 10^{-42}$
(cm³/s)

Ok, so if you want dark matter to be stable/non-annihilating/asymmetric...

make it fermionic?

Actually:

As it turns out, the attractive self-interaction cross-sections we covered earlier that fit galactic rotation curves imply neutron star bounds on DM fermions.

• Attractive Yukawa couplings allow fermions to overcome their fermi degeneracy pressure and collapse into black holes.

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The capture rate

$$C_X \sim 2.3 \times 10^{45} \text{ Gyr}^{-1} \left(\frac{\text{GeV}}{m_X}\right) \left(\frac{\rho_X}{10^3 \text{ GeV/cm}^3}\right)$$

Chandresekhar black hole collapse number for fermions and bosons

Fermions: $N_{chand} = 10^{57} (GeV/m_x)^3$ -need attractive S-I

Bosons: $N_{chand} = 10^{38} (GeV/m_x)^2$

- Attractive Yukawa couplings allow fermions to overcome their fermi degeneracy pressure and collapse into black holes.
- The current crop of models of self-interacting fermion dark matter for galactic halos will have implied annihilation interactions from old neutron stars.

- Attractive Yukawa couplings allow fermions to overcome their fermi degeneracy pressure and collapse into black holes.
- The current crop of models of self-interacting fermion dark matter for galactic halos will have implied annihilation interactions from old neutron stars.
- Calculations for fermions are more involved.
 - Collapse can occur from a (non) degenerate phase
 - Yukawa coupling can be (un)screened
 - Virial equation yields collapse conditions

- Assuming contact interactions, Fermions will scatter and thermalize the same as bosons.
- Fermions can be degenerate (or not)

$$E_{k,\text{deg.}} = \frac{(9\pi N_X/4)^{2/3}}{2m_X r^2}, \qquad E_{k,\text{non-deg.}} = \frac{3}{2}k_B T.$$

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Fermions can be screened (or not)

$$-2E_k + \frac{(\frac{4}{3}\pi)^{1/3}GN_X^{2/3}\rho_b m_X y^2}{m_\phi^2} + \frac{(\frac{4}{3}\pi)^{1/3}GN_X^{2/3}m_X^2 m_\phi}{y} + 8\alpha \left(\frac{m_\phi e^{-y}}{y} + m_\phi e^{-y}\right) = 0.$$

$$(\frac{4}{3}\pi)^{1/3}GN^{2/3}\alpha_b m_b y^2 - (\frac{4}{3}\pi)^{1/3}GN^{2/3}m_X^2 m_b - 4\pi\alpha m_b$$

$$-2E_k + \frac{(\frac{4}{3}\pi)^{1/3}GN_X^{2/3}\rho_b m_X y^2}{m_\phi^2} + \frac{(\frac{4}{3}\pi)^{1/3}GN_X^{2/3}m_X^2 m_\phi}{y} + \frac{4\pi\alpha m_\phi}{y^3} = 0.$$

Fermion Dark Matter Collapse Bound Channels						
State of	Degenerate,	Degenerate, Degenerate,		Non-degenerate,		
initial collapse	partly-screened	tly-screened strongly-screened strongly-screened		ly-screened		
Yukawa screening	$\alpha < m_{\phi}/m_X$	$\alpha > m_{\phi}/m_X$		-		
DM accum. #>	N_{acc} (Eqs. (4,5,7) >	N_{acc} (Eqs. (4,5,7)) >	Eqs	s.(4,7) >		
DM collapse #	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)			
Degenerate or	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)			
non-deg. collapse	$> 8 \times 10^{35}$	$> 8 \times 10^{35}$	$< 8 \times 10^{35}$			
State of second,	-	-	Partly-	Strongly-		
degenerate collapse	-	-	screened	screened		
Yukawa screening	-	-	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} < 1$	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} > 1$		
Continued	-	-	$\alpha > 1.6 \times 10^4$	$\alpha \ge 2.7 \times 10^{-9}$		
collapse	-	-	$\times \frac{m_{\phi}^2}{\sqrt{m_X^3 \text{GeV}}}$	$\times \frac{e^{2.1 \times 10^4 m_{\phi}/\sqrt{m_X(\text{GeV})}}}{(m_{\phi}/\text{GeV})}$		
Relativistic collapse	$\alpha > 4.7 m_{\phi}^2 / m_X^2$					
Star consumed	$N_{\text{coll.}} > \frac{3.4 \times 10^{36} \text{GeV}}{m_X}$					

State of	Degenerate,	Degenerate,	Non-degenerate,			
initial collapse	partly-screened	strongly-screened	strong	ly-screened		
Yukawa screening	$\alpha < m_{\phi}/m_X$	$\alpha > m_{\phi}/m_X$		-	Is it screened?	
DM accum. #>	N_{acc} (Eqs. (4,5,7) >	N_{acc} (Eqs. (4,5,7) > N_{acc} (Eqs. (4,5,7)) > Eqs.(4,7) >				
DM collapse #	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)			
Degenerate or	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)			
non-deg. collapse	$> 8 \times 10^{35}$	$> 8 \times 10^{35}$ $< 8 \times 10^{35}$		8×10^{35}		
State of second,	-	-	Partly-	Strongly-		
degenerate collapse	-	-	screened	screened		
Yukawa screening	-	-	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} < 1$	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} > 1$		
Continued	-	-	$\alpha > 1.6 \times 10^4$	$\alpha \ge 2.7 \times 10^{-9}$		
collapse	-	-	$\times \frac{m_{\phi}^2}{\sqrt{m_X^3 \text{GeV}}}$	$\times \frac{e^{2.1 \times 10^4 m_{\phi}/\sqrt{m_X(\text{GeV})}}}{(m_{\phi}/\text{GeV})}$		
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Star consumed		med $N_{\text{coll.}} > \frac{3.4 \times 10^{36} \text{GeV}}{m_X}$				

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initial collapse	partly-screened	strongly-screened strongly-screened		ly-screened	
Yukawa screening	$\alpha < m_{\phi}/m_X$	$\alpha > m_{\phi}/m_X$		-	Is it screened?
DM accum. # >	N_{acc} (Eqs. (4,5,7) >	$> N_{acc} \text{ (Eqs. } (4,5,7)) > \text{Eqs.} (4,7) >$			Satisfies virial
DM collapse #	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Ed	qs. (13),(23)	Collapse eq.?
Degenerate or	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)		
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State of second,	-	-	Partly-	Strongly-	
degenerate collapse	-	-	screened	screened	
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Continued	-	-	$\alpha > 1.6 \times 10^4$	$\alpha \ge 2.7 \times 10^{-9}$	
collapse	-	-	$\times \frac{m_{\phi}^2}{\sqrt{m_X^3 \text{GeV}}}$	$\times \frac{e^{2.1 \times 10^4 m_{\phi}/\sqrt{m_X(\text{GeV})}}}{(m_{\phi}/\text{GeV})}$	
Relativistic collapse					
Star consumed	tar consumed $N_{\text{coll.}} > \frac{3.4 \times 10^{36} \text{GeV}}{m_X}$				

State of	Degenerate,	Degenerate, Non-degenerate,				
initial collapse	partly-screened	strongly-screened	strongly-screened strongly-screened			
Yukawa screening	$\alpha < m_{\phi}/m_X$	$\alpha > m_{\phi}/m_X$		-	Is it screened?	
DM accum. # >	N_{acc} (Eqs. (4,5,7) >	N_{acc} (Eqs. (4,5,7)) >	Eqs	s.(4,7) >	Satisfies virial	
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Degenerate or	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Ed	qs. (13),(23)	Degenerate or	
non-deg. collapse	$> 8 \times 10^{35}$	$> 8 \times 10^{35}$	$< 8 \times 10^{35}$		Not?	
State of second,	-	-	Partly-	Strongly-		
degenerate collapse	-	-	screened	screened		
Yukawa screening	-	-	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} < 1$	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} > 1$		
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collapse	-	-	$\times \frac{m_{\phi}^2}{\sqrt{m_X^3 \text{GeV}}}$	$\times \frac{e^{2.1 \times 10^4 m_{\phi}/\sqrt{m_X(\text{GeV})}}}{(m_{\phi}/\text{GeV})}$		
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Fermion Dark Matter Collapse Bound Channels							
	State of	Degenerate,	Degenerate,	Non-c	legenerate,		
	initial collapse	partly-screened	strongly-screened	strong	ly-screened		
	Yukawa screening	$\alpha < m_{\phi}/m_X$	$\alpha > m_{\phi}/m_X$		-	Is it screened?	
	DM accum. # >	N_{acc} (Eqs. (4,5,7) >	N_{acc} (Eqs. (4,5,7)) >	Eqs	s.(4,7) >	Satisfies virial	
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	Degenerate or	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)		Degenerate or	
	non-deg. collapse	$> 8 \times 10^{35}$	$> 8 \times 10^{35}$	< 8	8×10^{35}	Not?	
	State of second,	-	-	Partly-	Strongly-	Collapses to	
	degenerate collapse	-	-	screened	screened	screened or not	
	Yukawa screening	-	-	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} < 1$	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} > 1$	degenerate state?	
	Continued	-	-	$\alpha > 1.6 \times 10^4$	$\alpha \ge 2.7 \times 10^{-9}$		
	collapse	-	-	$\times \frac{m_{\phi}^2}{\sqrt{m_X^3 \text{GeV}}}$	$\times \frac{e^{2.1 \times 10^4 m_{\phi} / \sqrt{m_X(\text{GeV})}}}{(m_{\phi}/\text{GeV})}$		
	Relativistic collapse						
	Star consumed	$N_{\rm coll.} > \frac{3.4 \times 10^{36} \text{GeV}}{1.00}$					

79

State of Degenerate,		Degenerate,	erate, Non-degenerate,		
initial collapse	partly-screened	strongly-screened	strong	ly-screened	
Yukawa screening	$\alpha < m_{\phi}/m_X$	$\alpha > m_{\phi}/m_X$		-	Is it screened?
DM accum. # >	N_{acc} (Eqs. (4,5,7) >	N_{acc} (Eqs. (4,5,7)) >	Eqs	s.(4,7) >	Satisfies virial
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Degenerate or	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)		Degenerate or
non-deg. collapse	$> 8 \times 10^{35}$	$> 8 \times 10^{35}$	< >	8×10^{35}	Not?
State of second,	-	-	Partly-	Strongly-	Collapses to
degenerate collapse	-	-	screened	screened	screened or not
Yukawa screening	-	-	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} < 1$	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} > 1$	degenerate state?
Continued	-	-	$\alpha > 1.6 \times 10^4$	$\alpha \ge 2.7 \times 10^{-9}$	Keeps collapsing
collapse	-	-	$\times \frac{m_{\phi}^2}{\sqrt{m_X^3 \text{GeV}}}$	$\times \frac{e^{2.1\times10^4 m_{\phi}/\sqrt{m_X(\text{GeV})}}}{(m_{\phi}/\text{GeV})}$	When degenerate?
Relativistic collapse					
Star consumed	$N_{\text{coll.}} > \frac{3.4 \times 10^{36} \text{GeV}}{m_{\text{V}}}$				80

80

Fermion Dark Matter Collapse Bound Channels						
	State of	Degenerate,	Degenerate,	Non-c	legenerate,	
	initial collapse	partly-screened	strongly-screened	strongly-screened strongly-screened		
	Yukawa screening	$\alpha < m_{\phi}/m_X$	$\alpha > m_{\phi}/m_X$		-	Is it screened?
	DM accum. # >	N_{acc} (Eqs. (4,5,7) >	N_{acc} (Eqs. (4,5,7)) >	Eqs	s.(4,7) >	Satisfies virial
	DM collapse #	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Ec	qs. (13),(23)	Collapse eq.?
	Degenerate or	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)		Degenerate or
	non-deg. collapse	$> 8 \times 10^{35}$	$> 8 \times 10^{35}$	$< 8 \times 10^{35}$		Not?
	State of second,	-	-	Partly-	Strongly-	Collapses to
	degenerate collapse	-	-	screened	screened	screened or not
	Yukawa screening	-	-	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} < 1$	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} > 1$	degenerate state?
	Continued	-	-	$\alpha > 1.6 \times 10^4$	$\alpha \ge 2.7 \times 10^{-9}$	Keeps collapsing
	collapse	-	-	$\times \frac{m_{\phi}^2}{\sqrt{m_X^3 \text{GeV}}}$	$\times \frac{e^{2.1 \times 10^4 m_{\phi} / \sqrt{m_X(\text{GeV})}}}{(m_{\phi}/\text{GeV})}$	When degenerate?
	Relativistic collapse		Rel. collapse?			
	Star consumed	$N_{\text{coll.}} > \frac{3.4 \times 10^{36} \text{GeV}}{m_X}$				81

Fermion Dark Matter Collapse Bound Channels					
State of	Degenerate,	Degenerate, Non-degenerate,		egenerate,	
initial collapse	partly-screened	strongly-screened	strongl	y-screened	
Yukawa screening	$\alpha < m_{\phi}/m_X$	$\alpha > m_{\phi}/m_X$		-	Is it screened?
DM accum. # >	N_{acc} (Eqs. (4,5,7) >	N_{acc} (Eqs. (4,5,7)) >	Eqs	.(4,7) >	Satisfies virial
DM collapse #	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)		Collapse eq.?
Degenerate or	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)		Degenerate or
non-deg. collapse	$> 8 \times 10^{35}$	$> 8 \times 10^{35}$	$< 8 \times 10^{35}$		Not?
State of second,	-	Neut	rayn	Strongly-	Collapses to
degenerate collapse	-		screened	screened	screened or not
Yukawa screening	- 04	tor D	$\frac{2 \times 10^4 m_{\phi}}{(2 \times 10^{1/2})^{1/2}} < 1$	$\frac{2 \times 10^4 m_{\phi}}{2 \times 10^{1/2}} > 1$	degenerate state?
Continued	-	tar B	0 > U × 10	$\gtrsim 2.7 \times 10^{-9}$	Keeps collapsing
collapse	-	-	$\times \frac{m_{\phi}^2}{\sqrt{m_X^3 \text{GeV}}}$	$\times \frac{e^{2.1 \times 10^4 m_{\phi} / \sqrt{m_X(\text{GeV})}}}{(m_{\phi}/\text{GeV})}$	When degenerate?
Relativistic collapse		Rel. collapse?			
Star consumed		82.			



Fermion Dark Matter Collapse Bound Channels						
State of	Degenerate, Degenerate,		Non-degenerate,			
initial collapse	partly-screened	strongly-screened	strong	ly-screened		
Yukawa screening	$\alpha < m_{\phi}/m_X$	$\alpha > m_{\phi}/m_X$		-		
DM accum. #>	N_{acc} (Eqs. (4,5,7) >	N_{acc} (Eqs. (4,5,7)) >	Eqs.(4,7) >			
DM collapse #	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Ed	qs. (13),(23)		
Degenerate or	Eq. (20)	Sol. Eqs. (12),(22)	Sol. Eqs. (13),(23)			
non-deg. collapse	$> 8 \times 10^{35}$	$> 8 \times 10^{35}$ $> 8 \times 10^{35}$ $< 8 \times 10^{35}$		8×10^{35}		
State of second,	-	-	Partly-	Strongly-		
degenerate collapse	-	-	screened	screened		
Yukawa screening	-	-	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} < 1$	$\frac{2 \times 10^4 m_{\phi}}{(m_X \text{GeV})^{1/2}} > 1$		
Continued	-	-	$\alpha > 1.6 \times 10^4$	$\alpha \gtrsim 2.7 \times 10^{-9}$		
collapse	-			$\times \frac{e^{2.1\times10^4 m_{\phi}/\sqrt{m_X({ m GeV})}}}{(m_{\phi}/{ m GeV})}$		
Relativistic collapse	$\alpha > 4.7m_{\phi}^2/m_X^2$					
Star consumed	$N_{\text{coll.}} > \frac{3.4 \times 10^{36} \text{GeV}}{m_X}$					

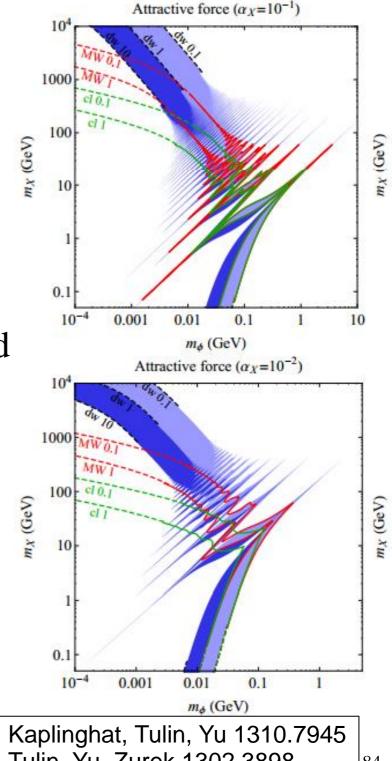
SIDM Fermions

Models which fit the cored dwarf galaxy profiles and are consistent with cluster and MW constraints will have attractive Yukawa potentials for the simplest scalar and pseudoscalar mediators.

$$V(r) = \pm \frac{\alpha_X}{r} e^{-m_{\phi}r}$$

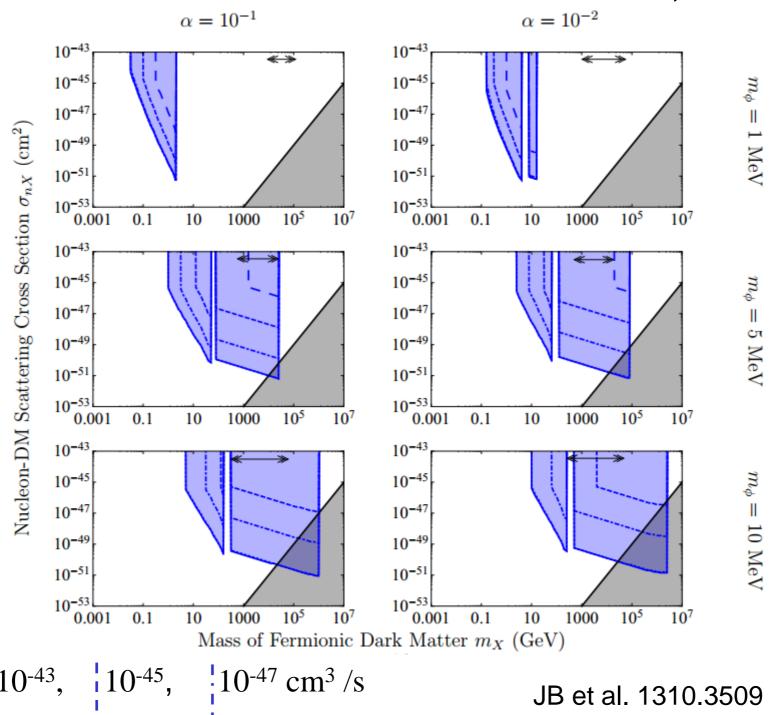
Light mediators give velocitydependent cross-sections.

$$\mathcal{L}_{\text{int}} = \begin{cases} g_X \bar{X} \gamma^{\mu} X \phi_{\mu} & \text{vector mediator} \\ g_X \bar{X} X \phi & \text{scalar mediator} \end{cases}$$



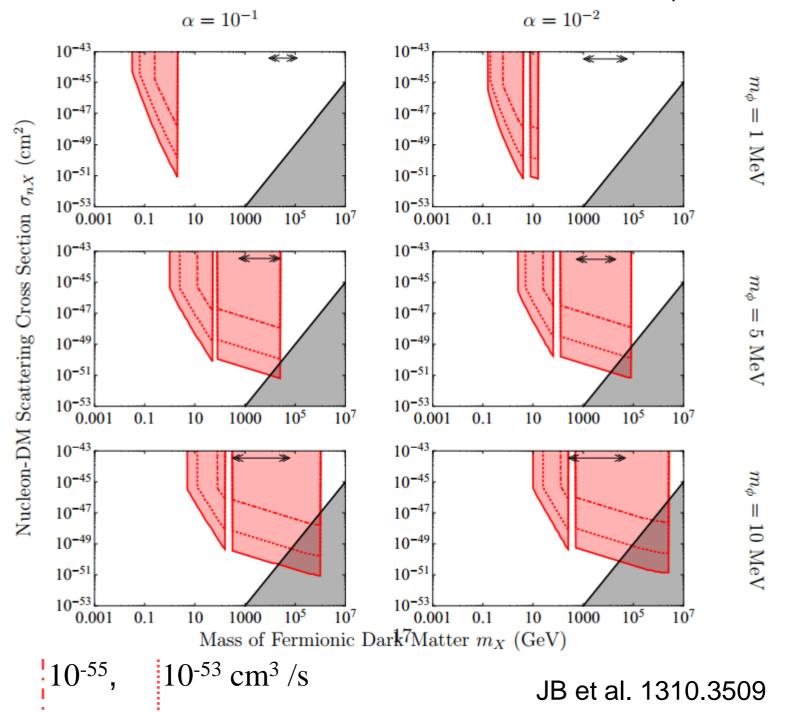
Tulin, Yu, Zurek 1302.3898

Bounds on SIDM Fermions, S-A



85

Bounds on SIDM Fermions, C-A



Final Thoughts

• Neutron stars serve as laboratories for establishing relationships between dark matter couplings.

• Future work: Apply EFT to interactions and compute scattering, thermalization, collapse for specific dark matter models – continue on with precision neutron

star bounds.

Thanks!

JB, Fukushima, Kumar 1301.0036
JB, Fukushima, Kumar, Stopnitzky 1310.3509
McDermott, Yu, Zurek 1103.5472
Kouvaris, Tinyakov 1104.0382
Guver, Erkoce, Reno, Sarcevic 1201.2400
Bell, Melatos, Petraki 1301.6811
Bertoni, Nelson, Reddy 1309.1721

Bonus Slides!

Self-Annihilation/Co-Annihilation

S-A is density dependent, 2nd order differential eq. that can be solved analytically.

$$N_{\rm acc}(t) \approx \sqrt{\frac{C_X V_{th}}{\langle \sigma_a v \rangle}} {\rm Tanh} \left[\sqrt{\frac{C_X \langle \sigma_a v \rangle}{V_{th}}} t \right], \qquad (t \le t_{non-deg.})$$

Exception: the DM is degenerate.

$$\frac{dN_{\rm acc}}{dt} \approx C_X - \frac{\sqrt{2} \langle \sigma_a v \rangle \left(G \rho_b m_X^2 \right)^{3/4} N_{acc}^{3/2}}{(3\pi)^{3/4}}. \qquad (t \ge t_{non.deg})$$

C-A provides a constant background

$$N_{\text{acc,coann.}} = \frac{C_X}{n_B \langle \sigma_a v \rangle_{co}} \left(1 - e^{-t_{ns} n_B \langle \sigma_a v \rangle_{co}} \right)$$

How old and how hot are neutron stars?

PSR J0437-4715

- -0.3 GeV/cm^3 local DM density
- 7 Gyr old
- 10⁶ K core temperature

Manchester et al. (2005)

